ANSWER KEY
Application of Quadratic Functions Worksheet
Problems to work out together in class

1. The value of a share of R. P. Mugs can be represented by $V(x) = x^2 - 6x + 13$, where x is the number of months after January 2001. What is the lowest value $V(x)$ will reach and when will this occur?

Answer: Since we are asked for the lowest value, we are looking for the minimum value of $V(x)$ and when that value will occur. We can find the minimum value by using the formula $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. Looking at $V(x)$, we know that $a = 1$ and $b = -6$. Plugging in these values we get the ordered pair (3, 4). This means that **3 months after January 2001, or April 2001, the value of a share of R. P. Mugs will reach its lowest value of $4.**

2. A farmer decides to enclose a rectangular garden, using the side of a barn as one side of the rectangle. What is the maximum area that the farmer can enclose with 40 feet of fence? What dimensions yield that maximum area?

Answer: Since the farmer is going to enclose a rectangular garden with 40 feet of fence, the farmer is looking at the perimeter of the garden. The perimeter of this garden is 40. Now we don't know what the length l and the width w. We do know that the sum of the 2 widths and the one length add up to the perimeter 40. (Note: we don't need another measure of length for the perimeter because we are using the side of a barn as one side of the garden. So, we have the formula $2w + l = 40.$

Now, we want to find the maximum area of this garden. Well, we have the formula $A = lw$ representing the area of the garden. We have two variables here; we would like our formula to only be in one variable. Well, we can solve for a variable in the perimeter equation and substitute it in. Solving for l gives, $l = 40 - 2w.$

After substituting this in to the area formula, we get $A(l) = (20 - 2w)w = -2w^2 + 40.w$

Since we want to find the maximum area, we can find the maximum of this quadratic function which will give us the maximum width. Letting $b = 40$ and $a = -2$, we get 10 to be the width. Plugging 10 into the equation $l = 40 - 2w$ gives $l = 20$. Thus our **maximum area is 200 feet^2.**

3. An envelope is 4 cm longer than it is wide. The area is 96 cm^2. Find the length & width.

Answer: Let *w* be the width of the envelope and let *l* be the length of the envelope. The first sentence says (mathematically) $l = w + 4.$ We also know that $96 = lw.$ Just as in the problem above, we can substitute our value of *l* from the first equation into our area equation. This gives $96 = (w+4)w = w^2 + 4w.$

Now, for this problem (unlike the others so far), all we are asked to do is solve for *w & l*.. We are not finding the maximum or minimum. So, we need to solve $96 = w^2 + 4w \Rightarrow w^2 + 4w - 96 = 0$ for *w*. We can factor this quadratic as $(w+12)(w-8) = 0$ and set each factor equal to zero and solve. Doing this gives *w* = 8 and -12. Since width can't be negative, **our envelope width is 8 feet. This then indicates that the length of the envelope is 12 feet**.

4. Sandy's car travels 280 miles averaging a certain speed. If the car had gone 5 mph faster, the trip would have taken 1 hour less. Find the average speed.

Answer: Using the formula *d=rt*, we have 280=*rt*. We also have 280=*(r+5)(t-1)*. Since the two distances are equal, we can set the two equations equal to each other to get *rt = (r+5)(t-1)*. Multiplying through gives $rt = rt - r + 5t - 5.$ Combining like terms gives $-r + 5t - 5 = 0.$

Again, we have two variables in one equation. As before, we will take one of the equations from above, solve for one of the variables, and substitute it in. Solving 280=*rt* for *t*, gives $t = \frac{280}{r}.$ Now, substituting this value of *t* in to $-r + 5t - 5 = 0$ gives $-r + 5\left(\frac{280}{r}\right) - 5 = 0.$

Now, we need to remove the value of *r* from the denominator. To do this, we can multiply both sides of the equation by *r*. This gives $-r^2 + 1400 - 5r = 0.$ We can now factor this quadratic as $-(r+40)(r-35) = 0$. Setting each factor equal to zero let's us **see that** $r = 35$ **mph.**

5. Three consecutive even integers are such that the square of the third is 76 more than the square of the second. Find the three integers.

Answer: Three consecutive even integers are represented by $x, x+2,$ and $x+4.$ From the question, we have the equation $(x+4)^2 = 76 + (x+2)^2.$ Multiplying through gives $x^2 + 8x + 16 = 76 + x^2 + 4x + 4 \Rightarrow x = 16.$ **So, our integers are 16, 18, and 20**.