DrDelMath

Important Properties of Quadratic Functions

Definition: A **quadratic function** is a function whose rule may be written in the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers and a is not zero.

Graph: The graph of a quadratic function is a parabola which opens up if $a > 0$ and opens down if $a < 0$.

Intercepts: The y-intercept of the graph of a quadratic function $f(x) = ax^2 + bx + c$ is (0, c).
The x-intercepts of the graph of a quadratic function $f(x) = ax^2 + bx + c$ are found by solving the corresponding quadratic equation in one variable, $ax^2 + bx + c = 0$.

Solving a Quadratic Equation in One Variable: Many quadratic equations in one variable may be solved by using factoring techniques in conjunction with the Zero Factor Property as described here:

1. Write the quadratic equation in standard form.
2. Factor the quadratic polynomial into a product of linear factors.
3. Use the Zero Factor Property to set each factor equal to zero.
4. Solve each of the resulting linear equations.

The resulting solutions are solutions of the original quadratic equation.

Solving a Quadratic Equation with The Quadratic Formula: The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant: The discriminant of a quadratic function $f(x) = ax^2 + bx + c$ is $b^2 - 4ac$.

1. If the discriminant is positive,the graph has two x-intercepts.
2. If the discriminant is zero,the graph has one x-intercept.
3. If the discriminant is negative,the graph has no x-intercepts and in this case;
   i. The graph is entirely above the x-axis if $a > 0$.
   ii. The graph is entirely below the x-axis if $a < 0$.

Vertex: The first coordinate of the vertex of the graph of a quadratic function $f(x) = ax^2 + bx + c$ is $-b/2a$. Because the vertex is on the graph of the function, the second coordinate is the range value associated with the first coordinate.
Therefore the vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Locating the Vertex: There are several convenient ways to locate the vertex of the graph of a quadratic function:

1. The vertex is on the line of symmetry of the parabola.
2. If the graph has two x-intercepts, the first coordinate of the vertex is midway between the two intercepts.

3. If the graph has one x-intercept, that intercept is the vertex.
4. The vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Graphing a Quadratic Function: To graph a quadratic function remember the graph of a quadratic function is a parabola which opens up if the leading coefficient is positive and opens down if the leading coefficient is negative. The x-intercepts are always important so they should be found, plotted, and labeled.
The x-intecepts of any function are found by finding the real zeros of the function.

The zeros of any function f are found by solving the equation resulting from f(x) = 0.

In the case of a quadratic function f the equation resulting from f(x) = 0 is always solvable with the Quadratic Formula or by factoring in conjunction with the Zero Factor Property.

The vertex of the parabola is also important and should always be found, plotted, and labeled.
The vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

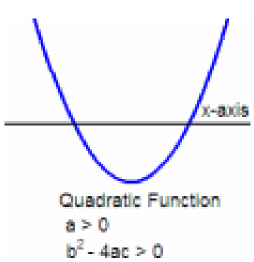
Law of Trichotomy and Quadratic Functions: Consider a quadratic function whose rule is written in the general form $f(x) = ax^2 + bx + c$.
From the Law of Trichotomy it follows that exactly one of the following is true about the leading coefficient:

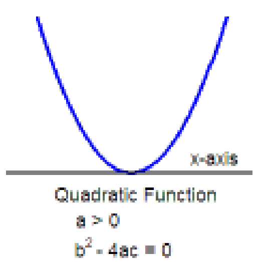
1. $a < 0$ and the parabola opens down.
2. $a = 0$ and the function is not quadratic.
3. $a > 0$ and the parabola opens up.

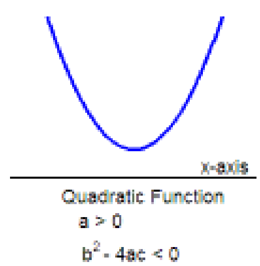
Also from Law of Trichotomy it follows that exactly one of the following statements about the discriminant is true:

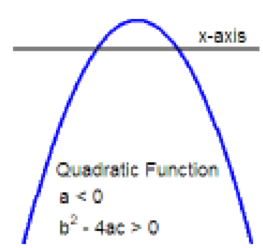
1. $b^2 - 4ac < 0$ and the parabola has no x-intercepts.
2. $b^2 - 4ac = 0$ and the parabola has exactly one x-intercept.
3. $b^2 - 4ac > 0$ and the parabola has two x-intercepts.

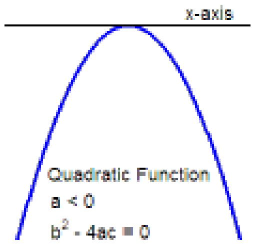
Pairing each of the two possibilities for the leading coefficient with each of the three possibilities for the discriminant, we find the following six graphical configurations to be the only possibilities for the graph of a quadratic function.

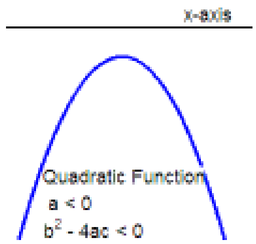









x-axis
Quadratic Function
a < 0
b2 - 4ac = 0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