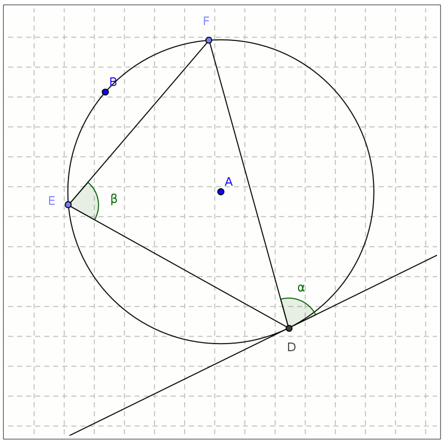
Proof of circle theorem 7

Alternate segment theorem

'The angle between the tangent and the chord at the point of contact is equal to the angle in the alternate segment'

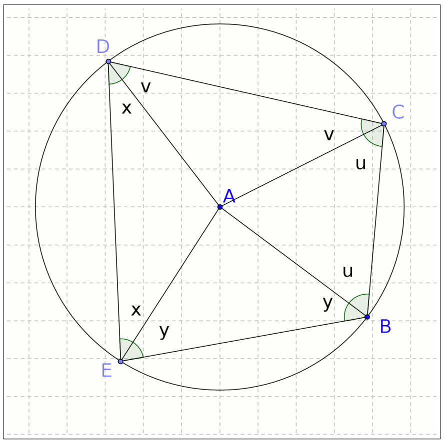


The chord DF divides the circle into two segments, and we're interested in the angle between this chord and the tangent at D, and the angle in the other (alternate) segment, E.

We need to show that D=E (see fig 1)

As before, the first step is to draw in radiuses from points on the circumference to the centre, A.

Fig 1



In the second diagram, fig 2, it is clear that the three triangles ADF, AFE and AED are isosceles, as a pair of sides in each triangle are radiuses.
Thus the two angles in ADF marked 'x' are equal (and similarly for y and z in the other triangles.)

Now, at D, $\alpha=90 - x$
(angle between radius and tangent is right angle)

In triangle DEF, $(x+y) + (y+z) + (z+x) = 180$
$2(x+y+z) = 180$
$x+y+z = 90$
but at E, $\beta = y+z = 90 - x = \alpha$
so $\beta = \alpha$

'The angle (α) between the tangent and the chord at the point of contact (D) is equal to the angle (β) in the alternate segment'

QED

Tim Devereux

(from http://www.timdevereux.co.uk/maths/geompages/proof7.php)