VECTORS 3D – Solutions

(1) (a) $|a| = \sqrt{9^2 + 2^2 + 6^2} = \sqrt{121} = \underline{11}$ (1)

(b) $|b| = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{49} = \underline{7}$ (1)

(c) $\underline{b} \cdot \underline{c} = 2\times2 + -6\times-1 + 3\times2 = \underline{16}$ (2)

(d) angle between $\underline{a}$ and $\underline{b}$ $= \dfrac{9\times2 + -2\times-6 + -6\times3}{11 \times 7}$

$\cos\theta = \dfrac{12}{77}$ (3)

$\underline{\theta = 81^\circ \text{ (nearest degree)}}$

(2) vector that multiples to 0 eg $\begin{pmatrix}2\\0\\-5\end{pmatrix}$ etc. (2)

(3) $\underline{\underline{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\-1\end{pmatrix}}$ (2)

(4) so $\begin{pmatrix}4\\-1\\12\end{pmatrix} = \begin{pmatrix}2\\3\\4\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\4\end{pmatrix}$

$4 = 2 + \lambda$ gives $\underline{\lambda = 2}$

$-1 = 3 - 2\lambda$ gives $\underline{\lambda = 2}$

$12 = 4 + 4\lambda$ gives $\underline{\lambda = 2}$

(4)

Same value of λ for all points so $\begin{pmatrix}4\\-1\\12\end{pmatrix}$ Does lie on Line

(5) $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}3\\1\\-5\end{pmatrix} + \lambda\begin{pmatrix}-1\\0\\2\end{pmatrix}$

$x = 3 - \lambda$

$y = 1$

$z = -5 + 2\lambda$

So Cartesian equation is (4)

$\underline{\underline{3 - x = \dfrac{z+5}{2} \text{ and } y = 1}}$

(6) If angle BAC a right angle then

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$$

$$\overrightarrow{AB} = B - A = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

$$\overrightarrow{AC} = C - A = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

and $\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = 0$

(5) So right angle

(7) $\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ -5 \\ -7 \end{pmatrix}$ at Intersection

$5 - 4\lambda = 1$ (1)

$1 + \lambda = 7 - 5\mu$ (2)

$2 - \lambda = 8 - 7\mu$ (3)

from (1) $\lambda = 1$

in (2) or (3) $\mu = 1$

So Intersection at $\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} + 1 \begin{pmatrix} -4 \\ 1 \\ -1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}}}$

or $\begin{pmatrix} 1 \\ 7 \\ 8 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ -5 \\ -7 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}}}$ (6)

out of 30.